

Table of Running Quark Mass Values : 1994

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Abstract

Running quark mass values $m_q(\mu)$ at some typical energy scales ($\mu = 1$ GeV, $\mu = m_W$, and so on) are reviewed. The values depend considerably on the value of $\Lambda_{\overline{MS}}$, especially, the value of top quark mass at $\mu = 1$ GeV does so. The relative ratios of light quark masses (m_u , m_d and m_s) to heavy quark masses (m_c , m_b and m_t) are still controversial.

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§1. Introduction

Recently, there has been considerable interest in phenomenological studies of quark and lepton mass matrices in order to obtain a clue to unified understanding of quarks and leptons. However, for this purpose, we must have the reliable knowledge of running quark mass values $m_q(\mu)$ which are evolved to an identical energy scale μ (e.g. $\mu = 1$ GeV). Since the earlier work by Gasser and Leutwyler [1], many works [2-5] on estimates of running quark masses have been reported. However, the values of $\Lambda_{\overline{MS}}$ which were adopted in these references [2-5] are not identical. Some of the input data have become older. On the other hand, this year (1994), the first observation [6] of top quark mass value has been reported, and the 1994 version of “Review of Particle Properties” (RPP94) [7] has been published. Therefore, this year is just timely for summarizing these works at present stage, and the review will be useful for physicists who intend to make a model-building of quarks and leptons.

In this review, we will give a summary table of running quark masses $m_q(\mu)$ at $\mu = 1$ GeV, $\mu = m_q$, $\mu = m_W$ and $\mu = \Lambda_W$, where $\mu = \Lambda_W$ is a symmetry breaking energy scale of the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$.

$$\Lambda_W \equiv \langle \phi^0 \rangle = (\sqrt{2}G_F)^{-\frac{1}{2}}/\sqrt{2} = 174 \text{ GeV} . \quad (1.1)$$

In this paper, we use the mass renormalization equation

$$\mu \frac{d}{d\mu} m_q(\mu) = -\gamma(\alpha_s) m_q(\mu) , \quad (1.2)$$

and do not use the renormalization equations for Yukawa couplings. This prescription is applicable only to the energy scale which is below the symmetry breaking energy scale Λ_W of the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$. If we want to evolve our results $m_q(\mu)$ to an extremely high energy scale far from $\mu = \Lambda_W$ (e.g. $\mu = \Lambda_{GUT}$), we must use the renormalization equations for Yukawa couplings.

In the next section, we review values of light quark masses $m_u(\mu)$, $m_d(\mu)$ and $m_s(\mu)$ at $\mu = 1$ GeV. In §3, we review values of heavy quark masses $m_c(\mu)$, $m_b(\mu)$ and $m_t(\mu)$ at $\mu = m_q$. In order to estimate $m_q(\mu)$ at any μ , we must know the values of the QCD parameters $\Lambda_{\overline{MS}}^{(n)}$ ($n=3,4,5,6$). In §4, the values of $\Lambda_{\overline{MS}}^{(n)}$ are evaluated. In §5, the values of $m_q(\mu)$ at $\mu = 1$ GeV, $\mu = m_q$, $\mu = m_w$ and $\mu = \Lambda_W$

are estimated. Finally, §6 is devoted to summary and discussion.

§2. Light quark masses

Grasser and Leutwyler [1] have concluded in their review article of 1982 that the light quark masses $m_u(\mu)$, $m_d(\mu)$ and $m_s(\mu)$ at $\mu = 1$ GeV are

$$\begin{aligned} m_u(1\text{GeV}) &= 5.1 \pm 1.5 \text{ MeV} , \\ m_d(1\text{GeV}) &= 8.9 \pm 2.6 \text{ MeV} , \\ m_s(1\text{GeV}) &= 175 \pm 55 \text{ MeV} . \end{aligned} \tag{2.1}$$

On 1987, Domingues and Rafael [2] have re-estimated those values. They have obtained the same ratios of the light quark masses with those estimated by Grasser and Leutwyler, but they have used a new value of $(m_u + m_d)$ at $\mu = 1$ GeV

$$(m_u + m_d)_{\mu=1\text{GeV}} = (15.5 \pm 2.0) \text{ MeV} , \tag{2.2}$$

instead of Grasser–Leutwyler’s value $(m_u + m_d)_{\mu=1\text{GeV}} = (14 \pm 4) \text{ MeV}$. Therefore, Dominguez and Rafael have concluded as

$$\begin{aligned} m_u(1\text{GeV}) &= 5.6 \pm 1.1 \text{ MeV} , \\ m_d(1\text{GeV}) &= 9.9 \pm 1.1 \text{ MeV} , \\ m_s(1\text{GeV}) &= 199 \pm 33 \text{ MeV} . \end{aligned} \tag{2.3}$$

Narison (1989) [3] has obtained

$$\begin{aligned} m_u(1\text{GeV}) &= 5.2 \pm 0.5 \text{ MeV} , \\ m_d(1\text{GeV}) &= 9.2 \pm 0.5 \text{ MeV} , \\ m_s(1\text{GeV}) &= 159.5 \pm 8.8 \text{ MeV} , \end{aligned} \tag{2.4}$$

by using $(m_u + m_d)_{\mu=1 \text{ GeV}} = (14.4 \pm 1.0) \text{ MeV}$.

On the other hand, Donoghue and Holstein (1992) [4] have estimated somewhat different quark mass ratios

$$\begin{aligned} r_1 &= (m_u + m_d)/[m_s + (m_u + m_d)/2] = 0.061 , \\ r_2 &= (m_d - m_u)/[m_s - (m_u + m_d)/2] = 0.036 . \end{aligned} \tag{2.5}$$

which lead to

$$m_d/m_u = 3.49, \quad m_s/m_d = 20.7 . \quad (2.6)$$

The value of m_d/m_u is considerably different from the previous values, e.g., Grasser–Leutwyler’s value $m_d/m_u = 1.75$. Donoghue and Holstein estimated the values (2.5) from the following four different sources: (1) $r_1 r_2 = 2.11 \times 10^{-3}$ from meson masses $+(\Delta m_R^2)_{EM}$, (2) $r_1 r_2 = 2.35 \times 10^{-3}$ from $\eta \rightarrow 3\pi$ decay, (3) $r_1/r_2 = 0.67 \pm 0.16$ from $\psi' \rightarrow J/\psi + \pi^0(\eta)$, and (4) $r_1 = 0.067 \pm 0.012$ from meson masses and L_7 . The values of r_1 and r_2 from these sources are still controversial.

Donoghue and Holstein’s value of m_s/m_d is in good agreement with that estimated by Dominquez and Rafael. Hereafter, we will adopt Dominguez-Rafael’s value (2.3) as light quark mass values at $\mu = 1$ GeV.

§3. Heavy quark masses at $\mu = m_q$

Pole mass

Sometimes, values of heavy quark masses m_c , m_b , and m_t are estimated in terms of the “pole” masses M_q^{pole} . It is known that the pole mass, $M_q^{\text{pole}}(p^2 = m_q^2)$, is a gauge-invariant, infrared-finite, renormalization-scheme-independent quantity.

Generally, mass function $M(p^2)$, which is defined by [1]

$$S(p) = Z(p^2) / (M(p^2) - \not{p}) , \quad (3.1)$$

$$Z(p^2) = 1 - \frac{\alpha_s}{3\pi} (a - 3b + \frac{2}{3}) \lambda + O(\alpha_s^2) , \quad (3.2)$$

is related to

$$M(p^2) = m(\mu) \left[1 + \frac{\alpha_s}{\pi} (a + \lambda b) + O(\alpha_s^2) \right] , \quad (3.3)$$

$$a = \frac{4}{3} - \ln \frac{m^2}{\mu^2} + \frac{m^2 - p^2}{p^2} \ln \frac{m^2 - p^2}{m^2} , \quad (3.4)$$

$$b = -\frac{m^2 - p^2}{3p^2} \left(1 + \frac{m^2}{p^2} \ln \frac{m^2 - p^2}{m^2} \right) , \quad (3.5)$$

where λ is given by $\lambda = 0$ in the Landau gauge and $\lambda = 1$ in the Feynman gauge. For $p^2 = m^2$, we obtain $a = 4/3$ and $b = 0$, so that we obtain the relation

$$M_q^{\text{pole}}(p^2 = m_q^2) = m_q(m_q) \left(1 + \frac{4}{3} \frac{\alpha_s}{\pi} + O(\alpha_s^2) \right) . \quad (3.6)$$

The estimate of the pole mass to two loops has been given by Gray *et al* [8]:

$$m_q(m_q) = M_q^{pole}(p^2 = m_q^2) \left[1 - \frac{4}{3} \frac{\alpha_s(M_q^{pole})}{\pi} - \left(K - \frac{16}{9} \right) \left(\frac{\alpha_s(M_q^{pole})}{\pi} \right)^2 + O(\alpha_s^3) \right] , \quad (3.7)$$

$$K = K_0 + \frac{4}{3} \sum_{i=1}^{n-1} \Delta(M_i^{pole}/M_n^{pole}) \simeq 17.15 - 1.04n + \frac{4}{3} \times 1.04 \sum_{i=1}^{n-1} \frac{M_i^{pole}}{M_q^{pole}} . \quad (3.8)$$

Here the sum in (3.8) is taken over $n - 1$ light quarks with masses M_i^{pole} ($M_i^{pole} < M_n^{pole} \equiv M_q^{pole}$). The exact expressions of K_0 and $\Delta(r)$ are given in Ref. [8]. The numerical values of $\Delta(M_i^{pole}/M_n^{pole})$ without approximation are tabled in Surguladze's paper [9]

Similarly, for the spacelike value of p^2 , $p^2 = -m_q^2$, we obtain $a = 4/3 - 2 \ln 2$ and $b = (2/3)(1 - \ln 2)$, so that we obtain the gauge-dependent ‘‘Euclidean’’ masses

$$M_q^{pole}(p^2 = -m_q^2) = m_q(m_q) \left[1 + \frac{\alpha_s}{\pi} \left(\frac{4}{3} - 2 \ln 2 \right) + O(\alpha_s^2) \right] . \quad (3.9)$$

Charm and bottom quark masses

Gasser and Leutwyler (1982) [1] have estimated charm and bottom quark masses m_c and m_b as

$$m_c(m_c) = 1.27 \pm 0.05 \text{ GeV} , \quad (3.10)$$

$$m_b(m_b) = 4.25 \pm 0.10 \text{ GeV} . \quad (3.11)$$

Narison (1989) [3] has, from ψ - and Υ -sum rules, estimated those as

$$M_c^{pole}(p^2 = -m_c^2) = 1.26 \pm 0.02 \text{ GeV} , \quad (3.12)$$

$$M_b^{pole}(p^2 = -m_b^2) = 4.23 \pm 0.05 \text{ GeV} , \quad (3.13)$$

which mean

$$M_c^{pole}(p^2 = m_c^2) = 1.45 \pm 0.05 \text{ GeV} , \quad (3.14)$$

$$M_b^{pole}(p^2 = m_b^2) = 4.67 \pm 0.10 \text{ GeV} , \quad (3.15)$$

with $\Lambda = 0.15 \pm 0.05 \text{ GeV}$.

Dominguez and Paver (1992) [5] have estimated the value of m_b as

$$M_b^{pole}(p^2 = m_b^2) = 4.72 \pm 0.05 \text{ GeV} , \quad (3.16)$$

from the ratio of Laplace transform QCD sum rules in the non-relativistic limit which is not so dependent on the value of Λ .

Recently, Tirard and Yuduráin [10] have re-estimated charm and bottom quark masses precisely and rigorously. They have concluded that

$$M_c^{pole}(p^2 = m_c^2) = 1.570 \pm 0.019 \mp 0.007 \text{ GeV} , \quad (3.17)$$

$$M_b^{pole}(p^2 = m_b^2) = 4.906_{-0.051}^{+0.069} \mp 0.004_{-0.040}^{+0.011} \text{ GeV} , \quad (3.18)$$

$$m_c(m_c) = 1.306_{-0.034}^{+0.021} \pm 0.006 \text{ GeV} , \quad (3.19)$$

$$m_b(m_b) = 4.397_{-0.002+0.004-0.032}^{+0.007-0.003+0.016} \text{ GeV} , \quad (3.20)$$

where the first- and second-errors come from the use of the QCD parameter $\Lambda_{\overline{MS}}^{(4)} = 0.20_{-0.06}^{+0.08} \text{ GeV}$ and the gluon condensate value $\langle \alpha_s G^2 \rangle = 0.042 \pm 0.020 \text{ GeV}^4$, and the third error denotes a systematic error. They have used $K_c \simeq 14.0$ and $K_b \simeq 13.4$ as the values of K_c and K_b given by (3.8).

Hereafter, we adopt Tirard and Yuduráin's values (3.19) and (3.20) as $m_c(m_c)$ and $m_b(m_b)$, although we do not adopt their value $\Lambda_{\overline{MS}}^{(4)} = 0.20 \text{ GeV}$ as $\Lambda_{\overline{MS}}^{(n)}$. For simplicity, we refer the values (3.19) and (3.20) as

$$m_c(m_c) = 1.306_{-0.035}^{+0.022} \text{ GeV} , \quad (3.21)$$

$$m_b(m_b) = 4.397_{-0.033}^{+0.018} \text{ GeV} . \quad (3.22)$$

Top quark mass

Recently, the CDF collaboration (1994) [6] has reported the top quark mass value

$$m_t = 174 \pm 10_{-12}^{+13} \text{ GeV} \quad (3.23)$$

from the data of $p\bar{p}$ collisions at $\sqrt{s} = 1.8 \text{ TeV}$. The value (3.23) is consistent with the recent standard-model-fitting value [11]

$$m_t = 161_{-16-22}^{+15+16} \text{ GeV} , \quad (3.24)$$

from LEP and $p\bar{p}$ collider data.

We adopt the value (3.23) as the top quark mass value at $\mu = m_t$. Hereafter, we will simply refer the value (3.23) as

$$m_t(m_t) = 174_{-27}^{+22} \text{ GeV} . \quad (3.25)$$

Note that usually the so-called standard-model-fitting value of m_t does not correspond to $m_t(m_t)$ but to $M_t^{pole}(p^2 = m_t^2)$. The CDF value of $m_t(m_t)$, (3.25), together with the value of $\Lambda_{\overline{MS}}^{(5)} = 0.195$ GeV [7] (see the next section), leads to

$$M_t^{pole}(p^2 = m_t^2) = 182_{-28}^{+23} \text{ GeV} . \quad (3.26)$$

§4. Estimates of the values of $\Lambda_{\overline{MS}}^{(n)}$

Prior to estimates of the running quark masses $m_q(\mu)$, we must estimate the values of $\Lambda_{\overline{MS}}^{(n)}$.

The effective QCD coupling $\alpha_s = g_s^2/4\pi$ is controlled by the β -function:

$$\mu \frac{\partial \alpha_s}{\partial \mu} = \beta(\alpha_s) , \quad (4.1)$$

where

$$\beta(\alpha_s) = -\frac{\beta_0}{2\pi}\alpha_s^2 - \frac{\beta_1}{4\pi^2}\alpha_s^3 + O(\alpha_s^4) , \quad (4.3)$$

$$\beta_0 = 11 - \frac{2}{3}n_q, \quad \beta_1 = 51 - \frac{19}{3}n_q , \quad (4.4)$$

and n_q is the effective number of quark flavors, so the $\alpha_s(\mu)$ is given by[†]

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0} \frac{1}{L} \left[1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln L}{L} + O(L^{-2} \ln^2 L) \right] . \quad (4.5)$$

where

$$L = \ln(\mu^2/\Lambda^2) . \quad (4.6)$$

At present, we can use only the expression of $\alpha_s(\mu)$ where the higher order term O in (4.5) is dropped. Then, the value of $\alpha_s(\mu)$ is not continuous at n th quark threshold μ_n (at which the n th quark flavor channel is opened), because the coefficients β_0 and β_1 in (4.2) depend on the effective quark flavor number n_q . Therefore, usually,

[†] In RPP94 [7] a three-loop expression of $\alpha_s(\mu)$ has been reviewed. However, at the moment, the two-loop expression (4.5) is sufficient for estimating running quark mass values to two-loops.

we use the expression $\alpha_s^{(n)}(\mu)$ (4.5) with a different $\Lambda_{\overline{MS}}^{(n)}$ for each energy scale range $\mu_n \leq \mu < \mu_{n+1}$, where $\Lambda_{\overline{MS}}^{(n)}$ are defined such as $\Lambda_{\overline{MS}}^{(n-1)}$ and $\Lambda_{\overline{MS}}^{(n)}$ satisfy the relation

$$\alpha_s^{(n-1)}(\mu_n) = \alpha_s^{(n)}(\mu_n) . \quad (4.7)$$

Therefore, we practically regard n th quark mass value $m_{qn}(\mu)$ at $\mu = m_{qn}$, $m_{qn}(m_{qn})$, as μ_n .

Particle data group (PDG) [7] has concluded that the world average value of $\Lambda_{\overline{MS}}^{(5)}$ is

$$\Lambda_{\overline{MS}}^{(5)} = 195_{-50}^{+65} \text{MeV} . \quad (4.8)$$

On the other hand, in the conventional quark mass estimates since Gasser- Leutwyler [1], the value $\Lambda_{\overline{MS}}^{(3)} = 150$ MeV is frequently used, although the value was used in the one-loop expression of $\alpha_s(\mu)$. For reference, we estimate $\Lambda_{\overline{MS}}^{(n)}$ and $m_q(\mu)$ for the case of $\Lambda_{\overline{MS}}^{(3)} = 150$ MeV as well as the case of $\Lambda_{\overline{MS}}^{(5)} = 195$ MeV.

Starting from $\Lambda_{\overline{MS}}^{(5)} \equiv 0.195$ GeV, by using the continuity condition of $\alpha_s(\mu)$, (4.7), at $\mu_5 = m_b(m_b) = 4.397$ GeV, $\mu_4 = m_c(m_c) = 1.306$ GeV, and $\mu_6 = m_t(m_t) = 174$ GeV, we obtain $\Lambda_{\overline{MS}}^{(4)} = 0.28475$ GeV, $\Lambda_{\overline{MS}}^{(3)} = 0.33156$ GeV and $\Lambda_{\overline{MS}}^{(6)} = 0.07760$ GeV. These results are summarized in Table IV.

Similarly, the values of $\Lambda_{\overline{MS}}^{(n)}$ are estimated for the case of $\Lambda_{\overline{MS}}^{(3)} \equiv 0.150$ GeV. The results are listed in Table IV.

Table IV. The values of $\Lambda_{\overline{MS}}^{(n)}$ in unit of GeV and $\alpha_s(\mu_n)$.

The underlined values are input values. Here, $\mu_4 = m_c(m_c) = 1.306$ GeV, $\mu_5 = m_b(m_b) = 4.397$ GeV, $\mu_6 = m_t(m_t) = 174$ GeV, and $m_Z = 91.187$ GeV are used.

	Case I	Case II
$\Lambda_{\overline{MS}}^{(3)}$	0.33156	<u>0.15000</u>
$\Lambda_{\overline{MS}}^{(4)}$	0.28475	0.11585
$\Lambda_{\overline{MS}}^{(5)}$	<u>0.19500</u>	0.07164
$\Lambda_{\overline{MS}}^{(6)}$	0.07760	0.02562
$\alpha_s(\mu_4)$	0.36122	0.23632
$\alpha_s(\mu_5)$	0.22122	0.16554
$\alpha_s(\mu_6)$	0.10539	0.09295
$\alpha_s(m_Z)$	0.11541	0.10606

§5. Estimates of running quark masses

The scale dependence of a running quark mass $\mu_q(\mu)$ is determined by the equation

$$\mu \frac{d}{d\mu} m_q(\mu) = -\gamma(\alpha_s) m_q(\mu) , \quad (5.1)$$

where

$$\gamma(\alpha_s) = \alpha_s \gamma_0 + \alpha_s^2 \gamma_1 + O(\alpha_s^3) , \quad (5.2)$$

$$\gamma_0 = 2 , \quad \gamma_1 = \frac{101}{12} - \frac{5}{18} n_q , \quad (5.3)$$

so that $m_q(\mu)$ is given by

$$m_q = \widetilde{m}_q \left(\frac{1}{2} L \right)^{-2\gamma_0/\beta_0} \left[1 - \frac{2\beta_1\gamma_0}{\beta_0^3} \frac{\ln L + 1}{L} + \frac{8\gamma_1}{\beta_0^2 L} + O(L^{-2} \ln^2 L) \right] , \quad (5.4)$$

where β_0 and β_1 are given in (4.3) and $L = \ln(\mu^2/\Lambda^2)$. Here, \widetilde{m}_q is the renormalization group invariant mass, which is independent of $\ln(\mu^2/\Lambda^2)$.

Since we interest only in the ratios $m_q(\mu)/\widetilde{m}_q$, we define the following quantity

$$R^{(n)} = \left(\frac{1}{2} L \right)^{-2\gamma_0/\beta_0} \left(1 - \frac{2\beta_1\gamma_0}{\beta_0^3} \frac{\ln L + 1}{L} + \frac{8\gamma_1}{\beta_0^2 L} \right) . \quad (5.5)$$

The value of $R^{(n)}$ is not continuous at $\mu = \mu_n$ (μ_n is the n th quark flavor threshold). Therefore, we calculate the evolution of the quark masses $m_q(\mu)$ from $\mu = \mu_A$ ($\mu_m \leq \mu_A < \mu_{m+1}$) to $\mu = \mu_B$ ($\mu_n \leq \mu_B < \mu_{n+1}$) as follows:

$$\frac{m_q(\mu_B)}{m_q(\mu_A)} = \left(\frac{R^{(m)}(\mu_{m+1})}{R^{(m)}(\mu_A)} \right) \left(\frac{R^{(m+1)}(\mu_{m+2})}{R^{(m+1)}(\mu_{m+1})} \right) \cdots \left(\frac{R^{(n-1)}(\mu_n)}{R^{(n-1)}(\mu_{n-1})} \right) \left(\frac{R^{(n)}(\mu_B)}{R^{(n)}(\mu_n)} \right) . \quad (5.6)$$

For example, the ratio $m_t(m_W)/m_t(1 \text{ GeV})$ is given by

$$\frac{m_t(m_W)}{m_t(1 \text{ GeV})} = \left(\frac{R^{(3)}(m_c)}{R^{(3)}(1 \text{ GeV})} \right) \left(\frac{R^{(4)}(m_b)}{R^{(4)}(m_c)} \right) \left(\frac{R^{(5)}(m_W)}{R^{(5)}(m_b)} \right) . \quad (5.7)$$

The values of $R^{(4)}(m_b)/R^{(4)}(m_c)$, $R^{(5)}(m_t)/R^{(5)}(m_b)$, and so on are summarized in Table V.

Table V. Values of $R^{(\mu)}(\mu)$ for the case I ($\Lambda_{\overline{MS}}^{(5)} = 0.195$ GeV) and the case II ($\Lambda_{\overline{MS}}^{(3)} = 0.150$ GeV).

	$\Lambda^{(5)} = 0.195$ GeV	$\Lambda^{(3)} = 0.150$ GeV
$R^{(3)}(1\text{GeV})$	1.00886 $\equiv 1$	0.738347 $\equiv 1$
$R^{(3)}(m_c)$	0.882993 0.87524	0.69043 0.93510
$R^{(4)}(m_c)$	0.84169 $\equiv 1$	0.64410 $\equiv 1$
$R^{(4)}(m_b)$	0.60363 0.71716	0.52206 0.81052
$R^{(5)}(m_b)$	0.55141 $\equiv 1$	0.47152 $\equiv 1$
$R^{(5)}(m_W)$	0.38415 0.69667	0.35418 0.75115
$R^{(5)}(m_t)$	0.36036 0.65353	0.33529 0.71111
$R^{(6)}(m_t)$	0.31051 $\equiv 1$	0.28700 $\equiv 1$
$R^{(6)}(\Lambda_W)$	0.31051 1.00000	0.28700 1.00000

In Table VI, we summarize the running quark mass values at $\mu = m_q$, $\mu = 1$ GeV, $\mu = m_W (= 80.22$ GeV) and $\mu = \Lambda_W (= 174$ GeV), where Λ_W is defined by

$$\Lambda_W \equiv \langle \phi^0 \rangle = (\sqrt{2}G_F)^{-\frac{1}{2}}/\sqrt{2} = 174 \text{ GeV} . \quad (5.8)$$

Table VI. Running quark mass values $m_q(\mu)$ (in unit of GeV) at $\mu = m_q$, $\mu = 1$ GeV, $\mu = m_W = 80.22$ GeV and $\mu = \Lambda_W = 174$ GeV. The upper values (lower values) are the running quark mass values in the case of $\Lambda^{(5)} \equiv 0.195$ GeV (the case of $\Lambda^{(3)} \equiv 0.150$ GeV).

	$m_q(m_q)$	$m_q(1\text{GeV})$	$m_q(m_W)$	$m_q(\Lambda_W)$
m_u	$0.3463^{+0.0017}_{-0.0018}$ ($0.1631^{+0.0015}_{-0.0017}$)	0.0056 ± 0.0011 (0.0056 ± 0.0011)	0.00245 ± 0.00048 (0.00319 ± 0.00063)	0.00230 ± 0.00045 (0.00302 ± 0.00059)
m_d	$0.3524^{+0.00013}_{-0.0015}$ (0.169 ± 0.019)	0.0099 ± 0.0011 (0.0099 ± 0.0011)	0.00433 ± 0.00048 (0.00564 ± 0.00063)	0.00406 ± 0.00045 (0.00534 ± 0.00059)
m_s	0.489 ± 0.021 (0.338 ± 0.029)	0.199 ± 0.033 (0.199 ± 0.033)	0.087 ± 0.014 (0.113 ± 0.019)	0.082 ± 0.014 (0.107 ± 0.018)
m_c	$1.306^{+0.022}_{-0.035}$ ($1.306^{+0.022}_{-0.035}$)	$1.492^{+0.023}_{-0.040}$ ($1.397^{+0.024}_{-0.037}$)	$0.653^{+0.009}_{-0.017}$ ($0.795^{+0.013}_{-0.021}$)	$0.612^{+0.010}_{-0.016}$ ($0.753^{+0.013}_{-0.023}$)
m_b	$4.397^{+0.018}_{-0.033}$ ($4.397^{+0.018}_{-0.033}$)	$7.005^{+0.029}_{-0.053}$ ($5.801^{+0.024}_{-0.044}$)	$3.063^{+0.013}_{-0.023}$ ($3.303^{+0.014}_{-0.025}$)	$2.874^{+0.012}_{-0.022}$ ($3.127^{+0.013}_{-0.023}$)
m_t	174^{+22}_{-27} (174^{+22}_{-27})	424^{+54}_{-66} (323^{+41}_{-50})	185^{+23}_{-29} (184^{+23}_{-29})	174^{+22}_{-27} (174^{+22}_{-27})

§6. Summary

We have estimated running quark mass values $m_q(\mu)$ at $\mu = m_q$, $\mu = 1$ GeV, $\mu = m_W = 80.22$ GeV and $\mu = \Lambda_W = 174$ GeV for the two cases, $\Lambda^{(5)} = 0.195$ GeV ($\Lambda^{(3)} = 0.332$ GeV, $\Lambda^{(4)} = 0.285$ GeV, $\Lambda^{(6)} = 0.0776$ GeV) and $\Lambda^{(3)} = 0.150$ GeV ($\Lambda^{(4)} = 0.116$ GeV, $\Lambda^{(5)} = 0.0716$ GeV, $\Lambda^{(6)} = 0.0256$ GeV). Of course, the case of $\Lambda^{(3)} = 0.150$ GeV has been listed only for reference, it is not our conclusion.

We have adopted the following quark mass values as the input values:

for light quark masses, Dominuez-Rafael's values:

$$\begin{aligned} m_u(1\text{GeV}) &= 5.6 \pm 1.1 \text{ MeV} , \\ m_d(1\text{GeV}) &= 9.9 \pm 1.1 \text{ MeV} , \\ m_s(1\text{GeV}) &= 199 \pm 33 \text{ MeV} , \end{aligned} \tag{2.3}$$

for charm and bottom quarks, Tirard and Yuduráin's values:

$$m_c(m_c) = 1.306^{+0.022}_{-0.035} \text{ GeV} , \tag{3.21}$$

$$m_b(m_b) = 4.397^{+0.018}_{-0.033} \text{ GeV} . \tag{3.22}$$

and, for top quark mass, CDF value:

$$m_t = 174^{+22}_{-27} \text{ GeV} . \tag{3.25}$$

The results are summarized in Table VI. As seen in Table VI, the running quark mass values (especially, those of heavy quarks at $\mu = 1 \text{ GeV}$, and those of light quarks at $\mu = m_W$ and $\mu = \Lambda_W$) are highly dependent on the value of $\Lambda_{\overline{MS}}$. The value of $\Lambda_{\overline{MS}}$ given in (4.8) includes large error values, so that the absolute values of quark masses in Table VI are not conclusive.

Although in Table VI, the values of $m_q(m_q)$ for light quarks are listed, those values, especially those for u and d , should not be taken rigidly, because $\alpha_s(\mu)$ rapidly increases at $\mu \leq m_s$, so that the perturbative result $R^{(n)}(\mu)$, (5.5), becomes unreliable in such a region.

The relative ratios among light quark masses at $\mu = 1 \text{ GeV}$ are fairly reliable, while the absolute values $m_q(1\text{GeV})$ are still controversial. The relative ratios of light quark masses to heavy quark masses may be somewhat changed in future.

In this paper, we have evaluated $m_q(\mu)$ only for energy scales μ which are below the electroweak symmetry breaking energy scale Λ_W . Running quark mass values at such an extremely high energy scale far from Λ_W will be given elsewhere.

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